Dirac cones in photonic crystal slabs

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Dirac cone can be materialized on the Γ point (Brillouin-zone center) of periodic photonic crystals and metamaterials by accidental degeneracy, as was shown by Huang et al. Then, Mei et al. discussed the formation of Dirac cones, Berry phase, and mapping into the Dirac Hamiltonian for phononic and photonic crystals by the \( k \cdot p \) perturbation theory. Because the Dirac point in the Brillouin-zone center is equivalent to a zero effective refractive index, it has much potential for various applications such as scatter-free waveguides and lenses of arbitrary shapes.

On the other hand, we showed by tight-binding approximation and group theory that Dirac cones can also be created by accidental degeneracy in the Brillouin-zone center of metamaterials, which are characterized by well-defined electromagnetic resonant states localized in their unit structures. We proved the presence of isotropic Dirac cones with auxiliary quadratic dispersion surfaces in square-, triangular-, and simple-cubic-lattice metamaterials [Fig. 1(a)] and the presence of the double Dirac cone, or a pair of identical Dirac cones, in the triangular-lattice metamaterials [Fig. 1(b)]. We also applied the \( k \cdot p \) perturbation theory and group theory to this problem and showed that the structure of the first-order perturbation matrix is determined almost uniquely by the mode symmetry, and completely clarified the conditions for obtaining the Dirac cones. We further analyzed the propagation of the Dirac cone modes in photonic crystal slabs and showed that their propagation direction can be controlled by the polarization of the incident plane wave.

In this presentation, we report on both the analytical and numerical calculations of the dispersion relation of photonic crystal slabs and present the sample parameters that materialize the two-dimensional Dirac cone. We also show that the dispersion relation for low-Q Dirac cone modes considerably deviates from the linear dependence [Fig. 2(b)] whereas the dispersion relation is linear for high-Q Dirac cone modes [Fig. 2(a)]. The deformed dispersion relation [Fig. 2(b)] apparently gives a superluminal propagation (\( v_g > c \)) of light waves.

Fig. 1 (a) Dirac cone with an auxiliary quadratic dispersion surface (dotted lines) and (b) double Dirac cone on the Γ point (\( k = 0 \)) of the two-dimensional Brillouin zone materialized by accidental degeneracy of two modes with particular combinations of mode symmetries.
These features (deformed dispersion and superluminal propagation) look similar to light propagation in anomalous dispersion media, which was extensively studied in 1980s. In fact, the apparent superluminal propagation is an artifact that was brought about by an insufficient treatment of the dispersion curves in which only the real part of the eigen frequency is taken into consideration. Indeed we have to properly consider the decrease of the electromagnetic energy during the propagation due to absorption and diffraction for the anomalous dispersion media and the photonic crystal slabs, respectively. We will present analytical results according to this prescription in the talk.

Fig. 2 The dispersion relation of the Dirac cone and an auxiliary quadratic mode in the vicinity of the $\Gamma$ point of the square-lattice photonic crystal slab. $a$: lattice constant, $t$: thickness, $R$: radius of air holes. (a) $t/a = 1.0$, (b) $t/a = 0.8$.

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References