An analogy and contrast between metamaterials and materials

Hiroharu Tamaru*
Photon Science Center, The University of Tokyo, Yayoi 2-11-16, Tokyo 113-8656, Japan
**E-mail: tamaru@psc.t.u-tokyo.ac.jp*

Metamaterial is conventionally defined as a collection of material structures whose characteristic length scale is much smaller than the wavelength of the electromagnetic wave under discussion. On the other hand, the term '*material* structures' implies the characteristic length to be sufficiently larger than the atomic or the molecular scale. Therefore, this definition itself leaves a very narrow or no window of acceptable size scale, if taken literally, for short wavelengths such as visible light.

The above definition was presumably worded in other to make a distinction between photonic crystals; or more naively, it is worded to be compatible with the definitions of permittivity and such in macroscopic electrodynamics in order to allow a straightforward introduction of the concept of homogenization. However, the popular definition of permittivity that employs an average of the response in a size scale that is 'sufficiently larger than atomic scale and sufficiently smaller than wavelength' itself find its difficulty for its interpretation in recent decades: Progresses in nano-scale science have shown that the continuum approximation is very often valid in practice for structures of sub-wavelength scales.

Switching to a phenomenological point of view, metamaterials and photonic crystals (and often, materials) can show many common phenomena: positive and negative refraction, high and low permittivity, low group velocity, energy confinement, etc. However, their formulation often differs (e.g. effective permittivity for metamaterials, band diagrams for photonic crystals, electronic band diagrams and joint density of states for materials), and their relation seems to be still under an active discussion.

In this talk, we discuss on these very conceptual parts of the framework. In order to enlighten the analogies between metamaterials, photonic crystals, materials, and so forth, we propose to shed more focus on the resonating nature of the 'structures,' and how they interfere in coherent propagation of energy.

We first focus on a coherent confinement of energy of some sort, and call it a resonator. A bound electron in quantum levels is one example, and light trapped in Fabry-Perot cavity could be another. Here, we disregard the form (electronic or photonic) of the energy confined, and how they are realized; we see them as an implementation detail. Resonators are generally attributed by its resonant frequency, and its interactions are discussed in terms of detuning with frequency of the 'media.'

Wave is a concept that realizes a medium that coherently propagates energy. We again disregard the form of the field and call it a propagator. Propagators are generally attributed by its wavevector, but, in abstract, it attributes the time delay between two positions during its propagation.

Under these views, a coherent system is a topological network of resonators connected by propagators.

Next, we focus on the conceptual layering of resonators and propagators (Figure 1). Take, for example, a two level electronic system like an atom or some small molecule, and represent it with a classical oscillator. The interaction of this resonator with incident light (propagator) is that the light field drives the oscillator, and it reemits light. The reemitted light always has a phase delay with respect to the light that just passed through. This delay can systematically be renormalized into a property of the propagator; in this case, permittivity. After the (conceptually) lower level resonator is renormalized into a propagator, it can be regarded as a propagator by itself. Then, the flow of the energy carried by this propagator can be fed back to itself by some sort of boundary (in

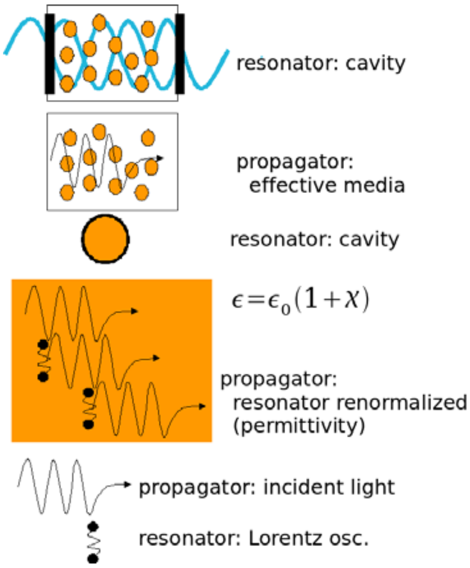

Figure 1: Conceptual layering of resonators and propagators, crossing electronic/photonic boundary.

this example, a spherical cavity), and the system can further be renormalized into a resonator (with a new resonance frequency).

Each of these procedures is nothing new, but we stress the conceptual importance of the alternate layering of resonators and propagators. Another aspect to note is that, in this picture, we seamlessly crossed the electronic/photonic boundary: The 'vehicle' of energy is not significant for the model at this stage, and is left as an implementation detail to derive concrete values.

With these simple views of the system, we propose yet anther definition of metamaterial that, in its wide sense, it is a collection of resonators coherently interacting with each other using propagators. The realized form of the resonators can classify them further; if they are all electronic, it is material, if all photonic, photonic crystals (or photonic structures, if they are not periodic), and if they are a combination of electronic and photonic resonators, it is metamaterial in its narrow sense. The conceptual layering of resonators and propagators leads to the analogies between different layers, and the layers they belong contrast each other.

With regards to photonic crystals, we point out that periodicity itself forms a resonating boundary, and Bloch's theorem is a (part of the whole) solution of the above mentioned interactions in this certain conditions. Crystals are of course no different, if we just switch the resonators and propagators to an electronic system.

Under this picture, a function of the system is the result of interferences of resonators at each layer. Many interesting phenomena are exhibited when two distinguished resonators in the same layer are strongly interacting. Photonic crystals carries those interesting phenomena in the optical frequencies, 'because its resonances are both at optical frequencies,' where as, for crystals, the resonance due to periodicity is usually far from resonance. Many metamaterials employ electronic resonators instead of the optical periodicity. This avoids Bragg scattering, and expands the path to deal the system 'more like materials,' but often brings in a complex spatial topology that make it anisotropic.

Some more aspects of the frame work will be given at the actual talk.

The author thanks Mr. Takuya Higuchi, Dr. Kei Sawada, and Dr. Kuniaki Konishi for many fruitful discussions. This work was supported in parts by KAKENHI on Innovative Areas 'Electromagnetic Metamaterials', KAKENHI on Priority Areas 'Strong Photon-Molecule Coupling Fields for Chemical Reactions', and by Photon Frontier Network Program from MEXT, Japan.